RHEODYNAMICS AND HEAT TRANSFER IN HOT COMPACTION OF POWDER MATERIALS

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A survey of results is presented from a theoretical investigation on the rheodynamics of powder materials in their high-temperature deformation.

For studying the processes of deformation of powder materials under SHS compaction and analyzing nonisothermal flow of compressible media in various zones of equipment rheodynamic models are used [1, 2], whose basic parameters are macroscopic density, velocity, and stresses in the material. These models are a complication of the earlier formulated thermal models [3, 4], describing the processes of cooling of combustion products and their heat exchange with the ambient medium.

The fundamental problem of theoretical consideration within the framework of rheodynamic models is finding the dependence of the density of a porous body on the applied pressure – the kinetics of its compaction. In doing this, it is important to answer a number of practical questions:

In what qualitatively different regimes does compaction of powder compressible media occur?

What is the influence of the thermal factor on the metal pressing?

In what cases is the material compacted but not extruded or, on the contrary, is extruded without compaction?

Of importance in the development of rheological models is the selection of rheological equations. We will note that powders of high-melting compounds are a special object, little investigated in the rheological context. In the theory of sintering and hot pressing the knowledge of such materials as of highly viscous liquids in the region of premelting temperatures is the most widespread [5].

The present work surveys the results of mathematical modeling of deformation and thermal processes of SHS compaction. Analytical solutions of the problem of one-sided compression and extrusion of powder materials are found which enable us at a qualitative level to establish different regimes of compaction and extrusion and to find criterial conditions for their realization. A numerical analysis of the influence of the nonuniformity of the thermal regime and the conditions of heat transfer on the regularities of compaction and extrusion of the material is performed.

1. Analytical Models of Hot Compaction of Powder Materials. Interest in the problem of one-sided compression of a viscous porous material was brought about both by the development of technology of hot-pressing methods and by the study of the high-temperature rheology of powder materials. At the initial stage it was considered under the assumption of the uniformity of density and the lack of friction on the walls [6]. The analytical solution obtained was used for selecting viscosimetric and rheological variables and creating methods for the determination of a priori unknown properties of the material and parameters of their dependence on density. Special features of the above-mentioned type of deformation with allowance for the nonuniformity of density distribution are investigated in [7]. As a result of a numerical solution of the problem the density, velocity, and normal stress profiles are determined for different regimes of hot pressing. The analytical solutions of the problem of one-sided compression of porous material made it possible to establish new regularities of the process, which expand the physical knowledge of it, and to reveal qualitatively different regimes of compaction – the regular regime and the wave one. A theoretical description for each of these regimes individually is given below.

Regular Regime of Compaction. For describing the selected type of flow of compressible powder material in the theory of hot pressing we used the system of continuity equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho V)}{\partial z} = 0, \tag{1}$$

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of the equations of motion

$$\rho\rho_1\left(\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial z}\right)=\frac{\partial\sigma_{zz}}{\partial z}$$

in combination with the rheological relations

$$\sigma_{zz}=\left(\frac{4}{3}\mu+\xi\right)\frac{\partial V}{\partial z},$$
$$\sigma_{rr}=\sigma_{\theta\theta}=\left(-\frac{2}{3}\mu+\xi\right)\frac{\partial V}{\partial z}. \quad (2)$$

At the initial instant the density distribution with the compact height is specified:

$$\rho(z,\ 0)=\rho_0(z). \quad (3)$$

At the lower boundary the condition of no-flow operates: $z = 0$: $V = 0$. At the upper boundary of the compact depending on the deformation conditions two types of boundary conditions should be distinguished: the regime with a specified force of the plunger of the press ($\sigma_{zz} = -P(t)$) and the regime with a specified velocity of movement of the plunger ($V = V(t)$).

We adopted the following empirical dependences of the shear μ and bulk ξ viscosities on density:

$$\mu(\rho)=\mu_1\rho^m,\ \ \xi(\rho)=\frac{4}{3}\mu(\rho)\frac{\rho}{1-\rho}. \quad (4)$$

We will note that in solving the system of equations considered one frequently disregards the inertial and nonstationary terms in the equation of motion and substitutes for these equations the simpler conditions of equilibrium

$$\frac{\partial\sigma_{zz}}{\partial z}=0. \quad (5)$$

One usually relates these assumptions to the smallness of the Reynolds number for the processes of hot compaction of high-melting powder materials, which is substantiated by the estimating calculations [7]. The problem was solved in the Lagrangian (q, t) coordinates, permitting us to obtain the analytical solutions and providing certain advantages in the interpretation of the results. The physical meaning of the Lagrangian coordinates is described in [8]. The kinetic equation of compaction

$$\frac{\partial\rho}{\partial t}=\frac{3}{4}\frac{P(t)}{\mu_1}\frac{1-\rho}{\rho^{m+1}}, \quad (6)$$

obtained on condition that the regime with the specified force of pressing is realized, may serve both for calculating the density distribution and for solving reverse problems, for example, for estimating the pressing time or viscosity of the solid base with the known density distribution. We will observe that, according to (6), the compaction rate with the initial nonuniform density distribution does not depend explicitly on the mass coordinate q and is presented in the form of the product of functions, one of which is dependent on time t, and the other – on density ρ. Exactly the same form of the kinetic equation is obtained in [6] for the case of a uniform density. Hence the important deductions of the character of the compaction process follow. Compaction of any separated individual volume of material with initial density ρ_0, for a nonuniform initial density distribution along the coordinate q, occurs in the same manner as for the uniform one, with the same initial density. By analogy with the thermal regular regime, this regime of compaction can be called regular. Taking the linear dependence of shear viscosity on density ($m = 1$) and the constant force on the plunger, we can obtain the following expression for the distribution of the material porosity $\Pi = 1 - \rho$:

$$\Pi(q,\ t)=\Pi_0(q)\exp(-t/t_*), \quad (7)$$

where $t_* = 4\mu_1/3P$ is the characteristic time of compaction. It can be seen that with time $t > t_*$ independently of $\Pi_0(q)$ the effect of self-equalization takes place, which was discussed in [7]. We will also point out that, according to (7), the characteristic time of compaction is equal for all individual volumes within the compressed material. This relation is convenient to use for determining the molding time of the material to the specified residual porosity.

Wave Regime of Compaction. Strictly speaking, the possibility of disregarding the inertial and nonstationary terms in the equation of motion is determined not only by the smallness of the number Re, but also by the value of partial derivatives of velocity with respect to the coordinates and time. At the same time it is inertness that determines a number of fundamental properties of the process. It is essential that for inertial media the disturbance from the plunger will propagate throughout the bulk of the material not instantaneously, thus creating prerequisites for forming a compaction wave in the porous medium. With the aim of investigating other regimes different from the regular one, we have found the solution of the problem of one-sided a compression in the form of compaction wave. The idea of intermediate asymptotics is used in this case. The compressible medium is considered infinite, and the boundary conditions are specified on $z = \pm\infty$. With $z = -\infty$, the material is stationary and is not compacted:

$$\rho(-\infty) = \rho_0, \quad V(-\infty) = 0.$$

With $z = +\infty$ we take the following conditions on the plunger: 1) for the regime with the specified velocity: $\rho(+\infty) = 1$, $V(+\infty) = V_p$; 2) for the regime with the specified force: $\rho(+\infty) = 1$, $\sigma_{zz}(+\infty) = P$.

Assuming the existence of a compaction wave moving across the material with a constant velocity $c = \text{const}$, and going to the system of coordinates which is related to the running wave [9], we determine the following profiles of density, velocity, and stresses in the material:

$$\rho(z) = \frac{\rho_0 + a\exp(\mathrm{Re}\,\xi)}{1 + a\exp(\mathrm{Re}\,\xi)},$$
$$V(z) = c + \frac{j}{k} = V_p \frac{a\exp(\mathrm{Re}\,\xi)}{\rho_0 \exp(\mathrm{Re}\,\xi)}, \tag{8}$$
$$\sigma(z) = \frac{\sigma_0 V_p a}{1-\rho} \frac{A\rho_0 \exp(\mathrm{Re}\,\xi)}{\rho_0^{+} \exp(\mathrm{Re}\,\xi)},$$

where

$$a = \frac{\rho_* - \rho_0}{1-\rho_*}; \quad A = \frac{\rho_1 c(1-\rho_0)}{\sigma_0}; \quad \xi = \frac{z}{H_0};$$
$$\rho_* = \rho(\xi = 0); \quad j = \rho(V - c); \quad \sigma_0 = 4\mu_1/3;$$

H_0 is the length of the product; j is the specific flow.

The work defines the boundaries of applicability of the constructed wave solution. It is obvious that the necessary prerequisite for realization of the compaction wave in a porous medium is the condition of smallness for the characteristic size of the wave as compared with the dimensions of the pressed material, i.e., $\delta < H_0$, where δ is the width of the wave front. Using (8) the width of the compaction wave front is expressible in terms of the characteristics of the compact (viscosity and density of the incompressible carcass) and the velocity of movement of the plunger:

$$\delta = -\frac{4}{3}\frac{H_0}{\mathrm{Re}} \ln \varepsilon^2 = -\frac{4}{3}\frac{\mu_1}{\rho_1 V_p} \ln \varepsilon^2,$$

where ε is a small quantity. It is evident that depending on the specific value of the Reynolds number the density profiles may have a qualitatively different form, and, consequently, it is possible to realize qualitatively different regimes of compaction: the regular regime (corresponding to a highly diffuse front of compaction) and the wave one, in which the compaction is localized in a narrow zone of compression (this regime corresponds to rather large Re numbers).

Taking into account the limiting character of both regimes, it is important to consider all varieties of regimes for compacting the porous powder mass, converted into a high-temperature state, as well as to investigate transient processes, realized in the intermediate, according to Reynolds, zone. The solution of these problems is possible within the framework of a generalized model, which includes the system (1) in its complete form, the boundary conditions (2), and the conditions at the compact boundary (3) [10]. The analysis of the results of numerical experiment made it possible to find criterial conditions for realizing various regimes of compaction of the hot porous mass. As a criterion determining the presence of this or that compaction regime we chose the generalized Reynolds number, the expression for which is obtained in [9]

$$\mathrm{Re}_*(P) = \frac{\sqrt{P\dfrac{1-\rho_0}{\rho_0\rho_1}}\,\rho_1 H_0}{\mu_1}.$$

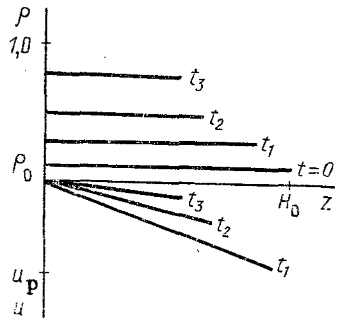

Fig. 1

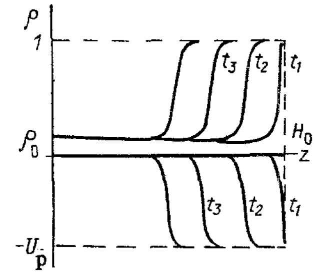

Fig. 2

Fig. 1. Density and velocity distribution with the height of the compact at various instants for the regular regime of compaction: $t_1 < t_2 < t_3$.

Fig. 2. Density and velocity distribution with the height of the compact for the wave regime of compaction: $t_1 < t_2 < t_3$.

It is actually a dimensionless complex, dependent on rheological and physical properties of the compressed material, dimensions of the compact, and technological parameters of the process, more convenient for analyzing the flow of the material with the specified force on the press. It is found that in the range $Re < Re_* = 1$ one of the limiting cases occurs – the regular regime of compaction, which corresponds to the linear form of the velocity profile and to simultaneous compaction of all individual volumes within the hot porous mass. The boundary value of Re_* corresponds to $Re \approx 1$. For example, at $\mu_1 = 10^7$ Pa·sec, $\rho_1 = 5 \cdot 10^3$ kg^3/m (which corresponds to high-melting metal powders converted into the high-temperature state), $P = 10^9$ Pa, $H_0 = 0.1$ m, the number $Re = 0.2 < Re_*$ (Fig. 1), which fits a pronounced regular regime in which the system has no time to feel the inertial factor since the time of hydrodynamic stabilization in this case is substantially smaller than the characteristic time of compaction: $t_h = H_0^2 \rho_1 / \mu_1 \sim 10^{-6} << t_* = 4\mu_1/3P \sim 10^{-3}$.

Another limiting case – compaction of the hot porous compact in the wave regime – is realized in the range $Re > Re_{**}$, starting with some lower boundary $Re_{**} = 25$. In the process of numerical calculation we can find the wave regime time, conditioned by the unsteadiness of the process: for instance, at $\mu_1 = 10^4$ Pa·sec, $\rho_1 \sim 10^3$ kg/m^3 (which is characteristic of polymer materials), $P = 10^{10}$ Pa, $H_0 = 0.1$ m ($Re \sim 45$) attaining the wave regime takes place for the time $t_y \sim 10^{-7}$ sec (Fig. 2). In this range for analyzing the processes of flow we could use the approximate wave solution of the problem given above.

The values of the Reynolds number, located between the boundary values Re_* and Re_{**}, are consistent with transient regimes of compaction. These regimes (Fig. 3) combine properties of the wave and regular regimes, but depending on the specific value of Re display them to a greater or lesser extent. If Re is close to Re_* then the transient regime is closer to the regular one in its properties. There exists a substantial feature, however, no effect of self-equalization of density from the outset of the process. There is more to be said – when the initial density throughout the volume of the material is equal, then in the transient regime a density gradient emerges during hydrodynamic stabilization. If Re is close to Re_{**}, then the transient regime is closer to the wave one, but with a rather diffuse, broad compaction wave front.

2. Analytical Model of Extrusion of Powder Materials. Models of plunger extrusion for incompressible ductile and viscous materials [11-15], which are usually considered for the majority of polymer and metal systems, are theoretically well-developed and investigated . However, these models often turn out to be inapplicable for describing the process of treatment by pressure of compressible powder high-melting compositions. The characteristics of rheological behavior of these materials, as well as a combination of the processes of compaction and extrusion, require a separate consideration for these systems. Investigated below is the process of extruding viscous compressible materials out of a chamber through the slot into a guide gauge. The model of the process formulated and the method employed for solving the problem in the Lagrangian coordinates enabled us to obtain an analytical solution of the problem and to find the distribution of density and velocity in the chamber and in the extruded rod.

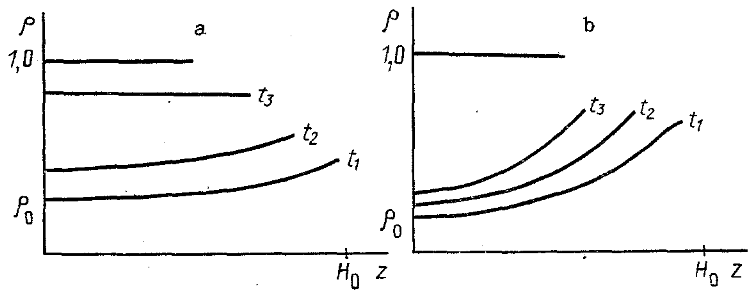

Fig. 3. Density distribution along the coordinate: a) Re → Re*; b) Re → Re**.

To describe the flow of the viscous porous material out of a cylindrical chamber, bounded at the top by a moving plunger, into the gauge, we use the system of equations (1)-(3), (5) with the following boundary and initial conditions [1]:

$$\sigma_{zz}|_{z=H(t)} = -P, \quad \rho(t)|_{t=0} = \rho_0(z), \tag{9}$$

where H(t) is the time dependence of the compact height. Extrusion of the material usually occurs into a die of conical shape, which is not taken into account in this consideration. This assumption is valid for a relatively short conical portion. Within the framework of the simultaneous approach the motion of the material is characterized by two parameters, the relative change in density and hydraulic resistance, dependent on the applied force:

$$\rho(0_-, t)/\rho(0_+, t) = B(|\sigma_{zz}(0)|),$$
$$-S_1\rho_1\rho(0_-, t)V(0_-, t) = f(|\sigma_{zz}(0)|).$$

The quantities B and f are defined by the die shape and rheological properties of the material. The work assumes B = 1, i.e., the absence of recompaction in the die. This approximation is acceptable when the major compaction takes place in the chamber. The dependence $f(|\sigma_{zz}(0)|)$ for simplicity is approximated by a power dependence: $f(|\sigma_{zz}(0)|) = k|\sigma_{zz}|^n$. The parameters k and n may be calculated from experimental data.

We found the analytical solutions which permitted us to perform the analysis of the limiting regimes of extruding the material without compaction and compacting the material with subsequent extrusion. The basic dimensionless parameter determining the process of compaction and extrusion is $\varkappa = t_{extr}/t_*$, which characterizes the relation of the extrusion time $t_{extr} = q_0/\bar{P}$, $\bar{P} = kP^nS_1/S_0\rho_1$, and the compaction time $t_* = 4\mu_1/3P$. From Fig. 4 it is seen, that at small values of $\varkappa$($\ln \varkappa << -1$) the specimen is extruded uncompacted ($\rho_{max} \simeq \rho_0$), and with $\ln \varkappa \geq 1.5$ compaction and then extrusion of the material occurs ($\rho_{max} \simeq 1$). With $-1 < \ln \varkappa < 1.5$ the processes of compaction and extrusion proceed in parallel. In essence, we have provided the answer to the question of a stage-by-stage consideration of the process of SHS extrusion. The stage-by-stage character of extrusion does not always take place. In practice this favorable situation should be maintained by various methods. For instance, this case is realized for a narrow slot with a large hydraulic resistance.

3. Nonisothermal Rheodynamics of SHS Pressing. The analytical solutions obtained made it possible to evaluate the conditions for realization of a quasiisothermal regime of compaction, in which the very process of pressing is not accompanied by a noticeable change of temperatures. However, in practice the nonuniformity of the thermal regime in the material and the conditions of heat transfer have a substantial effect on the distribution of densities, velocities, and stresses in the material, and consequently on the quality of the finished products. For studying the stress–strain state of the material under conditions of substantial nonisothermicity of the process we formulated a model of SHS compaction [2] which includes not only the equations of rheodynamics (1)-(3), (5) but also the heat-transfer equation

$$c\rho_1\left(\frac{\partial(\rho T)}{\partial t} + \frac{\partial(\rho V T)}{\partial z}\right) = \frac{\partial}{\partial z}\left(\lambda(\rho)\frac{\partial T}{\partial z}\right) - \frac{2\alpha}{r_0}(T - T_0) \tag{10}$$

with the additional boundary conditions for temperature

$$\lambda(\rho)\frac{\partial T}{\partial z} = \begin{cases}\alpha_1(T - T_0), & z = 0,\\ -\alpha_2(T - T_0), & z = H(t)\end{cases} \tag{11}$$

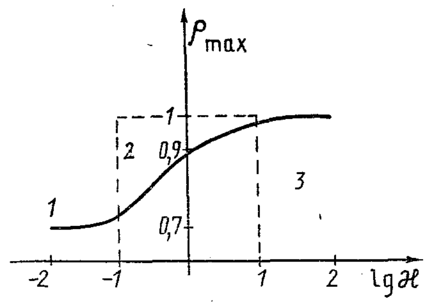

Fig. 4. Limiting density ρ_{max} as a function of ln ϰ: 1) region of extrusion of uncompacted material; 2) region of undercompaction; 3) extrusion of maximally compacted material.

and the initial temperature distribution in the specimen

$$T(z,\ 0) = T_0(z). \tag{12}$$

It should be noted that for the shear μ and bulk ξ viscosities we take account of their dependence not only on the density but also on the temperature:

$$\mu(\rho,\ T) = \mu_1(T)\mu_2(\rho) = \mu_1 \exp(U/RT)\rho^m,$$
$$\xi(\rho,\ T) = \frac{4}{3}\mu(\rho,\ T)\frac{\rho}{1-\rho} = \mu_1 \exp(U/RT)\frac{\rho^{m+1}}{1-\rho}. \tag{13}$$

To compare the results of numerical solutions with the analytical solution of the isothermal problem of one-sided compression obtained earlier in [8] and to simplify the initial system of equations (the equation of heat conduction in the Lagrangian system of coordinates is written in the convergent form) the problem was solved in the Lagrangian mass system of coordinates. The numerical calculations are in good agreement with the analytical solution of the isothermal problem only under adiabatic conditions [2]. If there is heat transfer between them, there is a considerable discrepancy. As an analysis of the numerical calculation showed, with nonisothermal pressing the following qualitatively different regimes of compaction are realized: 1) regime without compaction; 2) regime of maximal compaction; 3) regime of undercompaction. The decisive factors in the realization of one or another regime of pressing are the initial viscosity (at the combustion temperature) and the range of its change within the characteristic temperature interval (from the combustion temperature to the viable temperature).

The regime without compaction is realized when the viscosity of the solid base is sufficiently large. The critical value of viscosity above which this regime is realized is found by numerical calculations. It is the viscosity of the solid base or, in other words, the ductility of the grains of the solid carcass that is the decisive factor here. This factor determines the resistance to deformation in compaction, and consequently also the intensity of this process. We will note that in this regime the thermal processes and those of compaction weakly affect each other, i.e. proceed independently. In this case we can use the thermal model of SHS pressing [3] (without taking rheodynamic factors into account), proposed by the authors earlier. The regime of maximal compaction is realized when the initial viscosity of the solid base during the process always remains lower than the critical one, and changes slightly with temperature. In this case the processes of compaction do not depend on the temperature (on account of the dependence of the thermophysical properties on this variable). The regime of undercompaction is realized when the initial value of viscosity of the incompressible base of the material is lower than the critical one but due to the strong dependence on the temperature its value in the course of the process turns out to be higher than the critical value. Under the conditions of this regime the thermal processes and the processes of compaction proceed correlatively. Thus, variation of parameters of the temperature dependence of the solid base viscosity alone ensures a continuous transition from one regime of compaction to another.

In [7-8] it is noted that in the case of isothermal compaction of compacts with a nonuniform initial density distribution there is an effect of self-equalization of density along the height of the compact. The presence of heat losses, on the contrary, often contributes to a substantial increase of the initial heterodensity in the material; this effect increases as the viscosity increases and the heat insulation of less dense layers of the material remains weak.

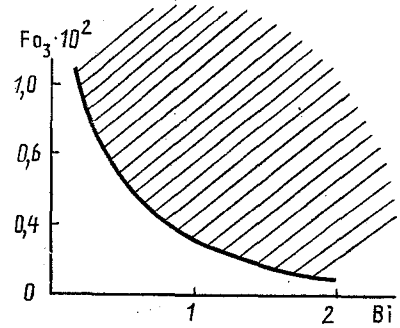

Fig. 5. Region of producing a compact product ($\rho > 0.9$) in the plane of the parameters Fourier delay (Fo_3)–Biot (Bi).

An important technological parameter of SHS compaction is the delay time (the time from the onset of the initiation of the chemical reaction to the pressure supply). The effect of this parameter on the distribution of density and stresses in the material is studied. It is shown that as the delay time increases due to strong cooling of the material from the ends the difference of radial stresses over the specimen increases, and an ever increasing portion of the material near its ends remains undercompacted; the maximal density of the material also becomes substantially lower.

The limiting value of density and the time the material remains in the plastic state depends on the conditions of heat transfer (Biot numbers) and on the technological parameters. An analysis of numerical results shows that the condition of existence of the compaction regime is some boundary curve Fo_3–Bi (Fo_3 is the delay time in the dimensionless form), below which lies the working region of SHS pressing, and above which the material turns out to be undercompacted (Fig. 5).

4. Nonisothermal Rheodynamics of Extrusion of Compressible Materials. The analytical solution obtained in [1] for the problem of extrusion of a material out of the chamber took no account of the nonisothermicity of the process of SHS extrusion. In practice, thermal nonisothermicity leads to a nonuniform compaction, and in the final analysis to the deterioration of properties of the product obtained.

To describe flow of the material out of a cylindrical mold to a guide gauge we will use the system of equations of continuity and equilibrium together with the rheological relations (1)-(3), (5) and the boundary and initial conditions (9). The unknown relative density ρ and velocity V here are functions solely of the coordinate z and time t. The dependence of shear and bulk viscosities on density and temperature is taken into account [13].

For studying the effect of the thermal factor on the stress–strain state of the material we added to the system of equations the equations of heat conduction for the material (the subscript i = 1 in the chamber, i = 2 in the gauge).

$$c\rho_1\left[\frac{\partial(\rho T_i)}{\partial t}+\frac{\partial(\rho V T_i)}{\partial z}\right]=\frac{\partial}{\partial z}\left(\lambda(\rho)\frac{\partial T_i}{\partial z}\right)-\frac{2\alpha_i}{r_i}(T_i-T_0)$$

with the boundary and initial conditions

$$z=H(t):\lambda(\rho)\frac{\partial T_1}{\partial z}=-\alpha_4(T_1-T_0),\quad z=-L(t):\lambda(\rho)\frac{\partial T_2}{\partial z}=\alpha_3(T_2-T_0),$$

$$z=0:T_1=T_2,\quad J_2=\psi J_1,\quad J_i=-\lambda(\rho)\frac{\partial T_i}{\partial z}\quad (i=1,\ 2),$$

$$T_1(z,\ 0)=T_2(z,\ 0)=T_*(z).$$

Here we assume that the temperature across the section z = const of the compact is constant due to its small transverse dimension in comparison with the length and the thermophysical properties of the material do not depend on the temperature. Heat removal in the transverse direction is taken into account by the last terms of the equations of heat conduction.

The movement of the upper H(t) and lower L(t) boundaries of the specimen is taken into account in the model by the equations

$$\frac{dH(t)}{dt}=V_p(t),\quad \frac{d(-L(t))}{dt}=V(0_-,\ t),$$

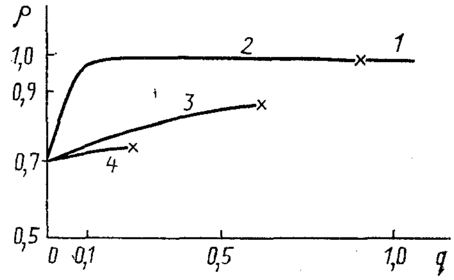

Fig. 6. Density distribution ρ along the mass coordinate q in a rod for various delay times: 1) $t_d = 0$; 2) 5 sec; 3) 10 sec; 4) 15 sec.

$V(0_-, t)$ is the velocity of the material at z = 0 in the rod:

$$V(0_-, t) = -\frac{kP^n}{\rho_1 \rho(0, t)}.$$

To make a direct comparison with the analytical solution of the isothermal problem of extrusion and to simplify the system of equations (to write the equations of heat conduction in divergent form), and also to reduce the number of transient boundaries (instead of two transient boundaries of the region, i.e., the upper H(t) and lower L(t) boundaries, we obtain one transient boundary, corresponding to the die hole z = 0), we will go over to the Lagrangian mass system of coordinates [1].

The system of equations was reduced to a dimensionless form and solved numerically using the conservative balance schema, i.e., the schema ensuring accurate (without regard to a round-off error) consistency with the laws of conservation on any grid in the finite region, containing an arbitrary number of nodes of the difference grid [16].

As a result of the numerical solution of the problem we establish the distribution of temperature, density, velocity, and stresses for the material in the mold and for the rod, produced by extrusion, at any instant. The end of the calculation was either the condition of attaining the temperature in the die hole, which is lower than the viable temperature (the temperature, above which the material retains its ductility and below which it freezes) or the complete extrusion of the material, i.e., attaining a value close to unity by the transient boundary.

As the calculations show, good agreement of the time dependences of density, obtained analytically and numerically, takes place solely under adiabatic conditions. If there is heat transfer between these dependences, there is a substantial discrepancy.

Earlier, on [17] it was shown that there exist three different regimes of extrusion: the quasistationary regimes of extrusion without compaction and of maximally compacted material, and the intermediate regime. Realization of one or another regime was dependent on the parameter $\varkappa$, determined by the relation of the characteristic times of compaction and extrusion. Under the nonisothermal conditions of deformation of the material the thermal factors affect the characteristic time of compaction in terms of viscous properties and their temperature dependences. Varying the temperature regime before pressing in one way or another, one should provide such a level or rheological properties which would produce the optimal relation between the characteristic times of compaction and extrusion.

One of the determining parameters in selecting the technological regime of forming the product is the delay time. Figure 6 shows the density distribution with the height of an extruded rod for various delay times. Curve 4 (Fig. 6) corresponds to the regime without compaction revealed in [17], curves 1, 2 – to the regime of maximal compaction of the end portion of the specimen, curve 3 – to the regime of undercompaction along the entire length of the specimen.

Within the framework of the present model, in contrast to the isothermal consideration of the process of SHS extrusion, according to the analytical model [1], it is possible to describe the conditions for realization of the regime of clogging of the die outlet. The condition of clogging was determined by the viable temperature of the material above the die outlet, much as it was performed in the thermal model (see [3, 4]). However, in practice, what matters is not only whether the clogging will happen or not but also what density distribution along the length of the extruded portion depending on the delay time is. Curve 1 (Fig. 6) is consistent with the case when the entire rod has the density >0.99 excluding its small portion (less than 5%). It is precisely this density distribution that is the most favorable in terms of practice. As the delay time increases up to 5 sec, a portion of the material is not extruded; however, the density of the larger portion of the extruded material is close to unity. As the delay time increases further the portion of the extruded material decreases (curves 3, 4, Fig. 6), and the porosity of the rod itself is high.

NOTATION

t, time; r, z, transverse and longitudinal coordinates; ρ, relative density of the material; ρ_1, μ_1, density and viscosity of the incompressible base of the material; μ, ξ, shear and bulk viscosities of the material; σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, radial, tangential, and axial stresses; S_0, S_1, cross-sectional areas of the chamber and the gauge; P, force on the plunger of the press; q_0, relative mass of the compact; $\varkappa$, dimensionless parameter, i.e., the relation of the characteristic times of compaction and extrusion; V, velocity of flow of the material; K_1, n, experimental parameters of hydraulic resistance of the die hole; ρ_{max}, maximal value of relative density; $\rho_0(q)$, initial density distribution throughout the bulk of the compact; α_3, α_4, heat-transfer coefficients at the lower and upper ends of the compact; $\psi = S_1/S_0$, degree of deformation.

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